



6.1820/MAS.453: Mobile and Sensor Computing aka IoT Systems

<https://6mobile.github.io/>

Lecture 9: Introduction to Inertial Sensing & Sensor Fusion

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Some material adapted from Gordon Wetzstein (Stanford) and Sam Madden (MIT)

Announcements

Lab 2 due today; Lab 3 out

Soon will share project ideas and inventory list. Start forming teams!

How is IoT used in war, defense...?

- GPS
 - originally military project
 - drones, missiles...
 - other location technologies: UWB
- IMUs
 - also location, navigation
- Networking:
 - Ad hoc networks (gov shutdown internet)
 - LEO satellites (spaceX)
- Sensors on combatants
 - Location, vital signs
- AR/VR
- Seeing through walls
- Underwater IoT

Situation awareness and coordination are cornerstones of modern warfare.
Sensors and connectivity are core to these capabilities

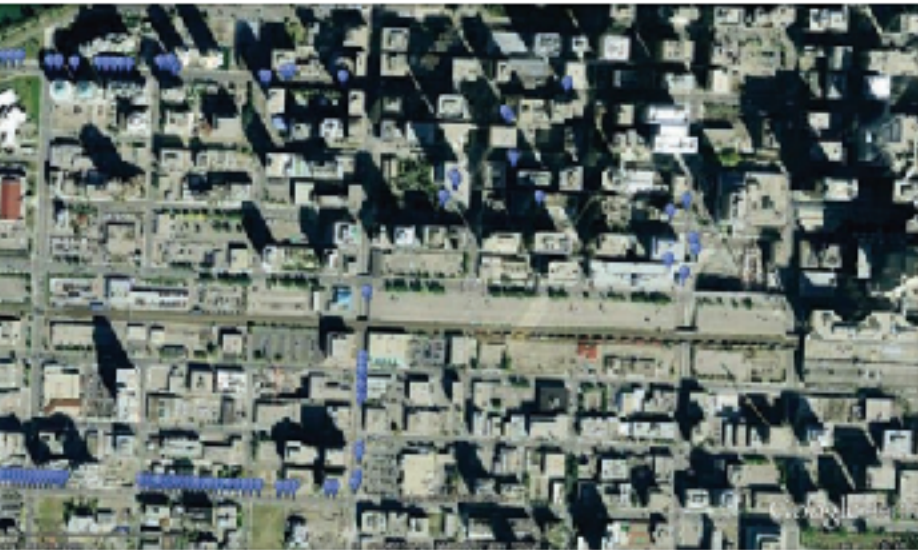
Overview: The Next Two Lectures

Fundamentals & Applications of Inertial Sensing

this lecture

1. What inertial sensing modalities are relevant for location inference?
2. How can we leverage physics and mathematics fundamentals to build reliable, microscopic sensors?
3. Case-study based application of inertial sensing:
Pothole patrol
4. Practical approaches to accounting for sensory noise in real-world settings.

Example Application: Inertial Navigation



GPS only

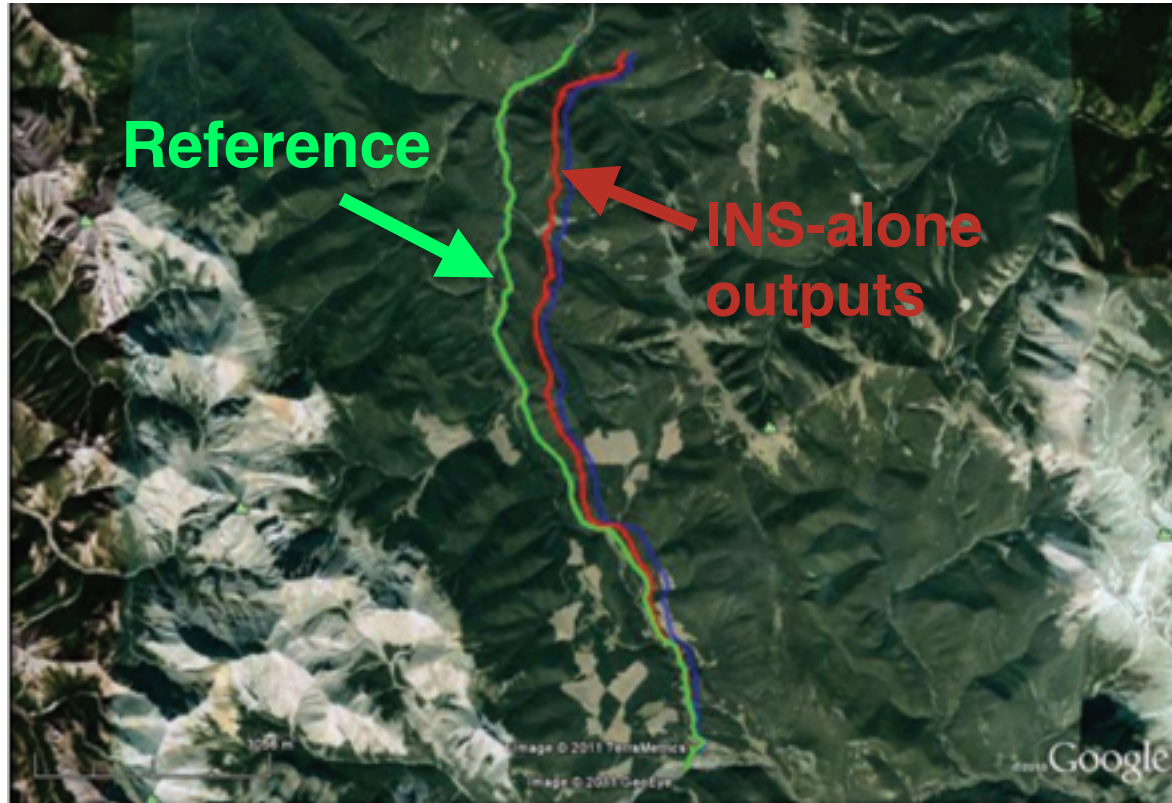


GPS+INS

Key Idea #1: Integrate acceleration data over time to discover location (Inertial Sensing)

Inertial Sensing alone is not enough for accurate positioning

- Errors accumulate over time



Source: INS Face Off
MEMS versus FOGs

Key Idea #2: Fuse Data from Multiple Sensors
(Sensor Fusion)

This Lecture

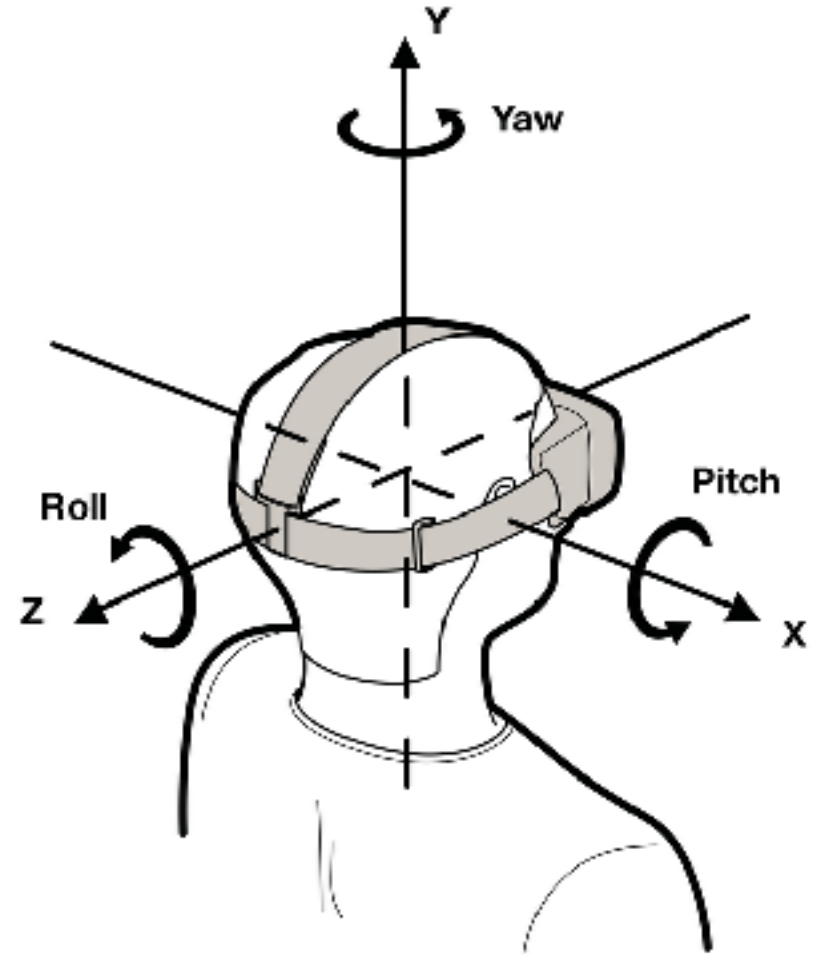
Key Idea #1: Integrate acceleration data over time to discover location (Inertial Sensing)

Key Idea #2: Fuse Data from Multiple Sensors (Sensor Fusion)

What are the 6 degrees of freedom in localization?

Let's understand inertial sensing in the context of VR

- **Goal:** track location and orientation of head or other device
- **Coordinates:** Six degrees of freedom:
 - Cartesian frame of reference (x,y,z)
 - Rotations represented by Euler angles (yaw, pitch roll)



What does an IMU consist of?
(Inertial Measurement Unit)

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- **Gyroscope** measures angular velocity ω in degrees/s
- **Accelerometer** measures linear acceleration \mathbf{a} in m/s^2
- **Magnetometer** measures magnetic field strength \mathbf{m} in μT (micro-Teslas).

Why is it called IMU?

History of IMUs

- Earliest use of gyroscopes goes back to German ballistic missiles (V-2 rocket) in WW2 for stability



- In the 1950s, MIT played a central role in the development of IMUs (Instrumentation Lab)

Where are IMUs used today?

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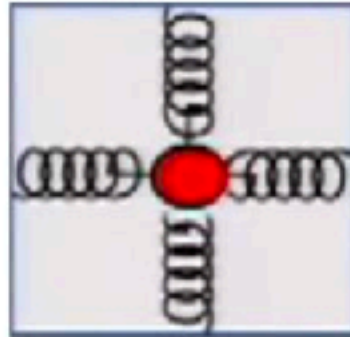


Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- Understanding Sources of Errors
- Dead reckoning by fusing multiple sensors

Accelerometer

Mass on spring



Gravity

$$1g = 9.8\text{m/s}^2$$

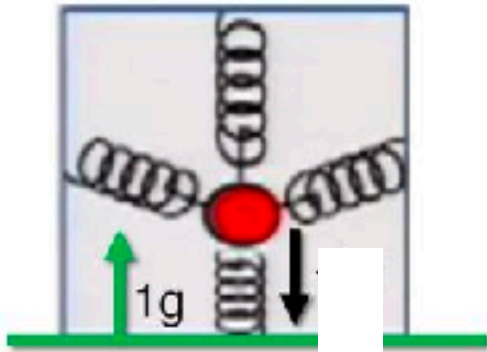
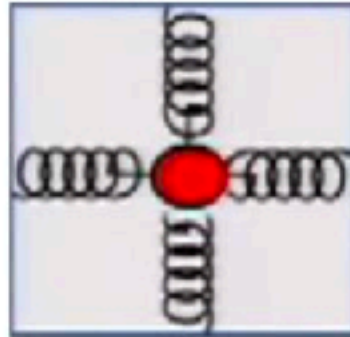
Free Fall

Linear Acceleration

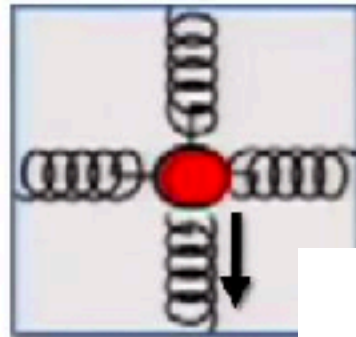
Linear
Acceleration
plus gravity

Accelerometer

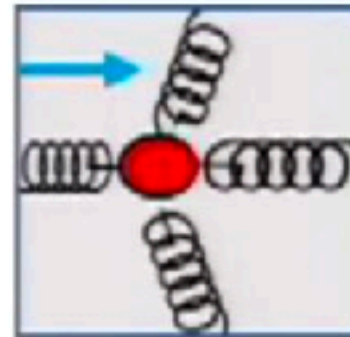
Mass on spring



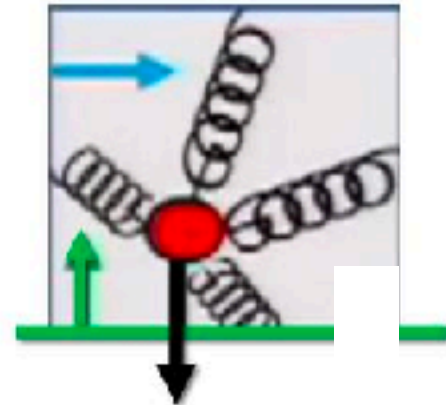
Gravity
 $1g = 9.8m/s^2$



Free Fall

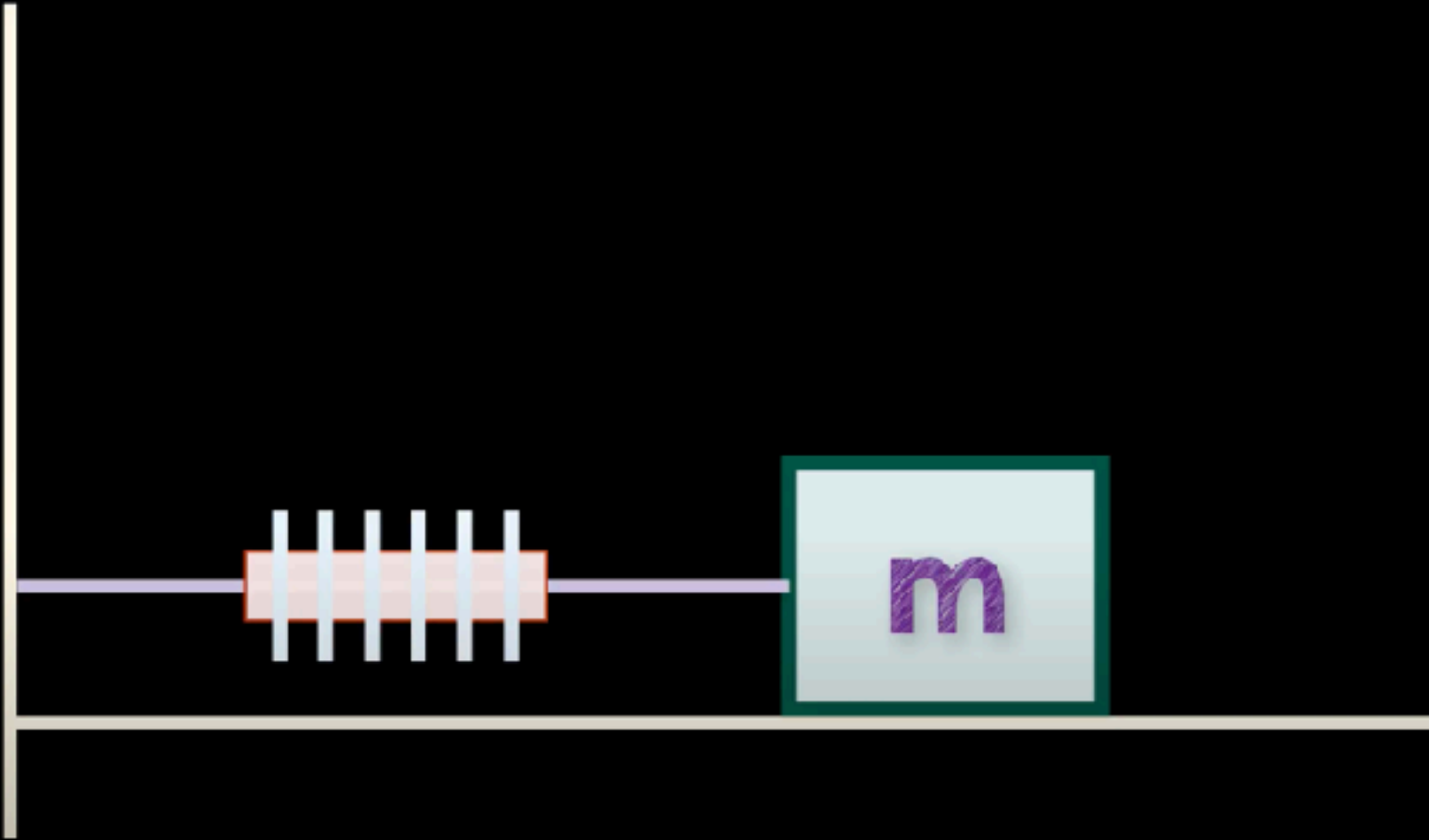


Linear Acceleration



Linear
Acceleration
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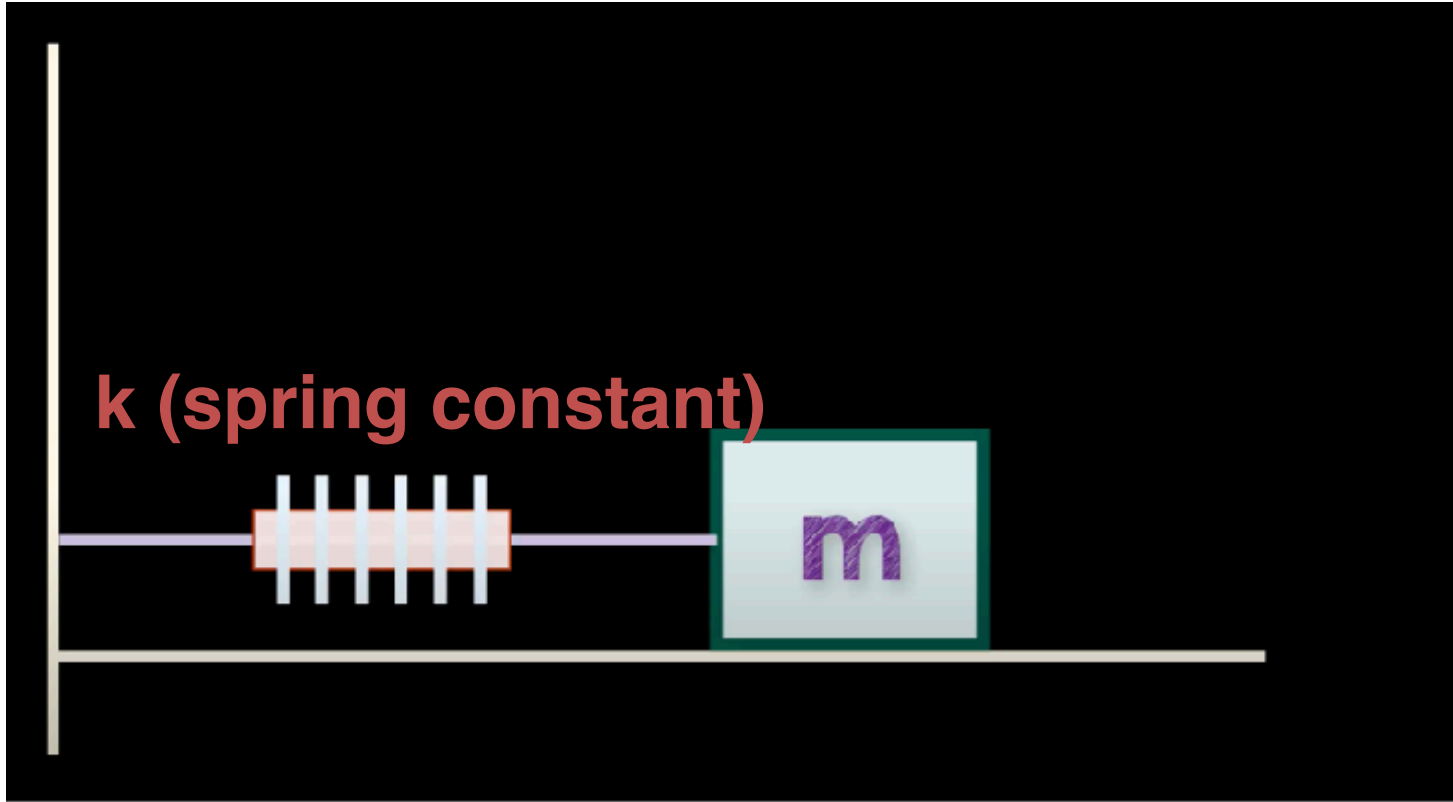
How Accelerometers Work



What matters is the displacement

k (spring constant)





Hooke's Law

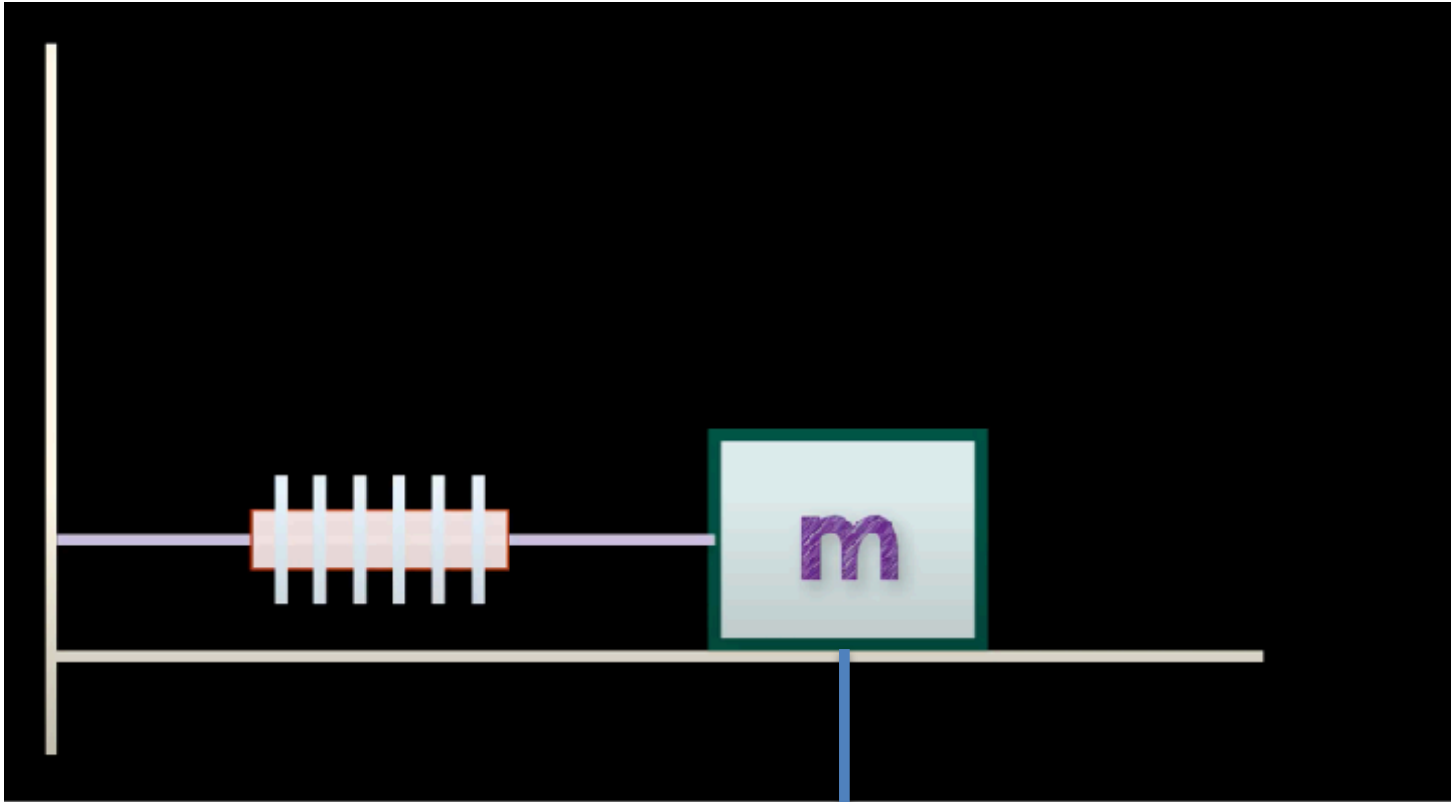
$$F = kx$$

Newton's Law

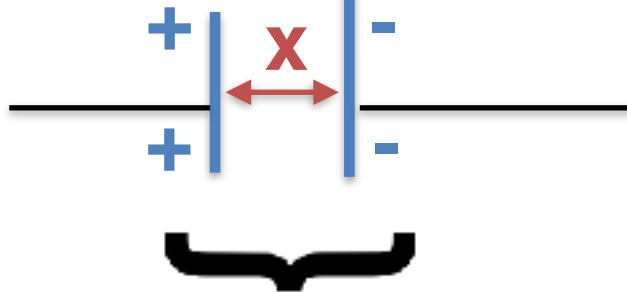
$$F = ma$$

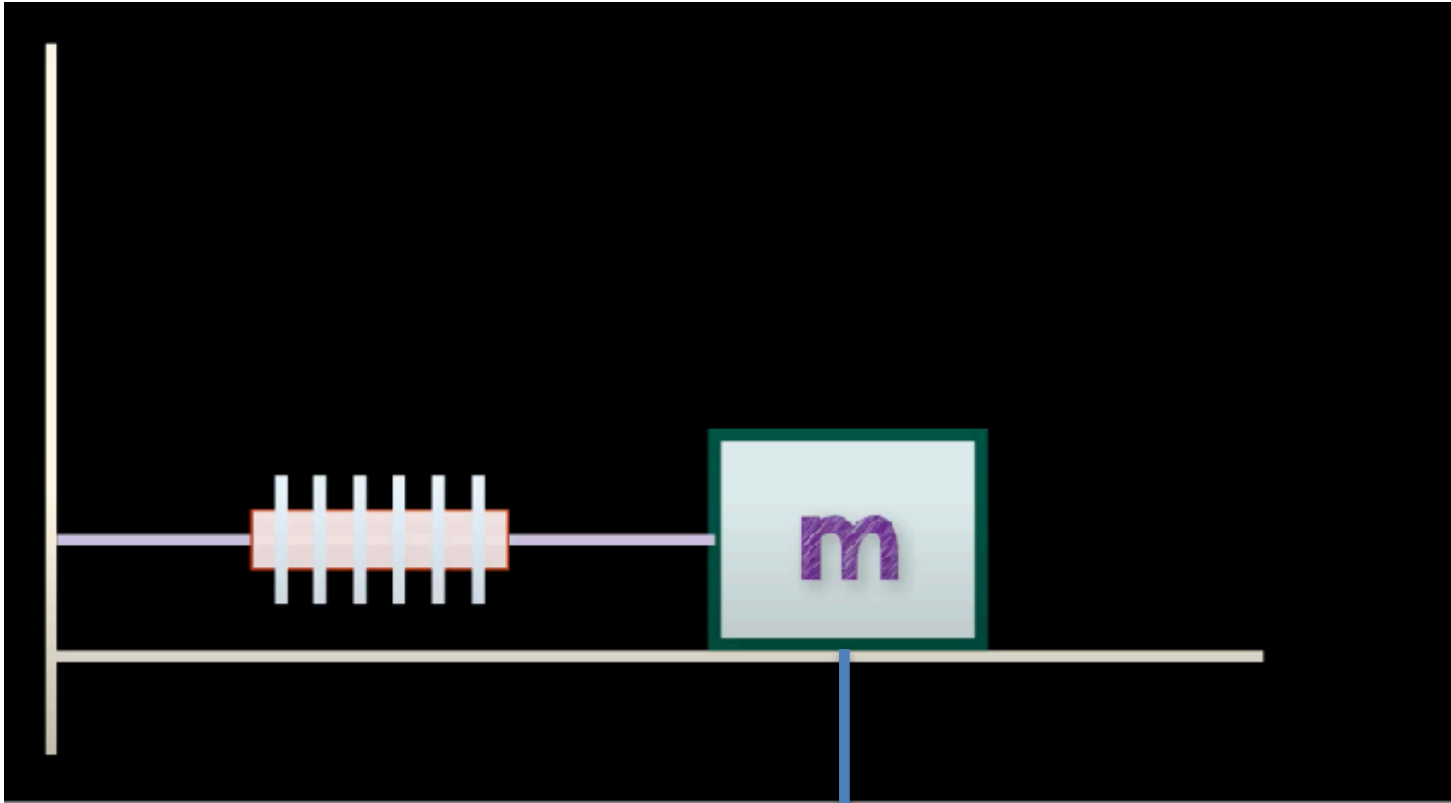
$$\Rightarrow a = \frac{k}{m}x$$

Why not simply use displacement to measure displacement?

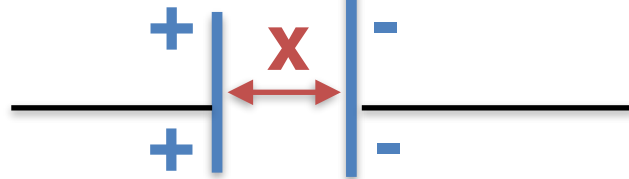


Capacitor





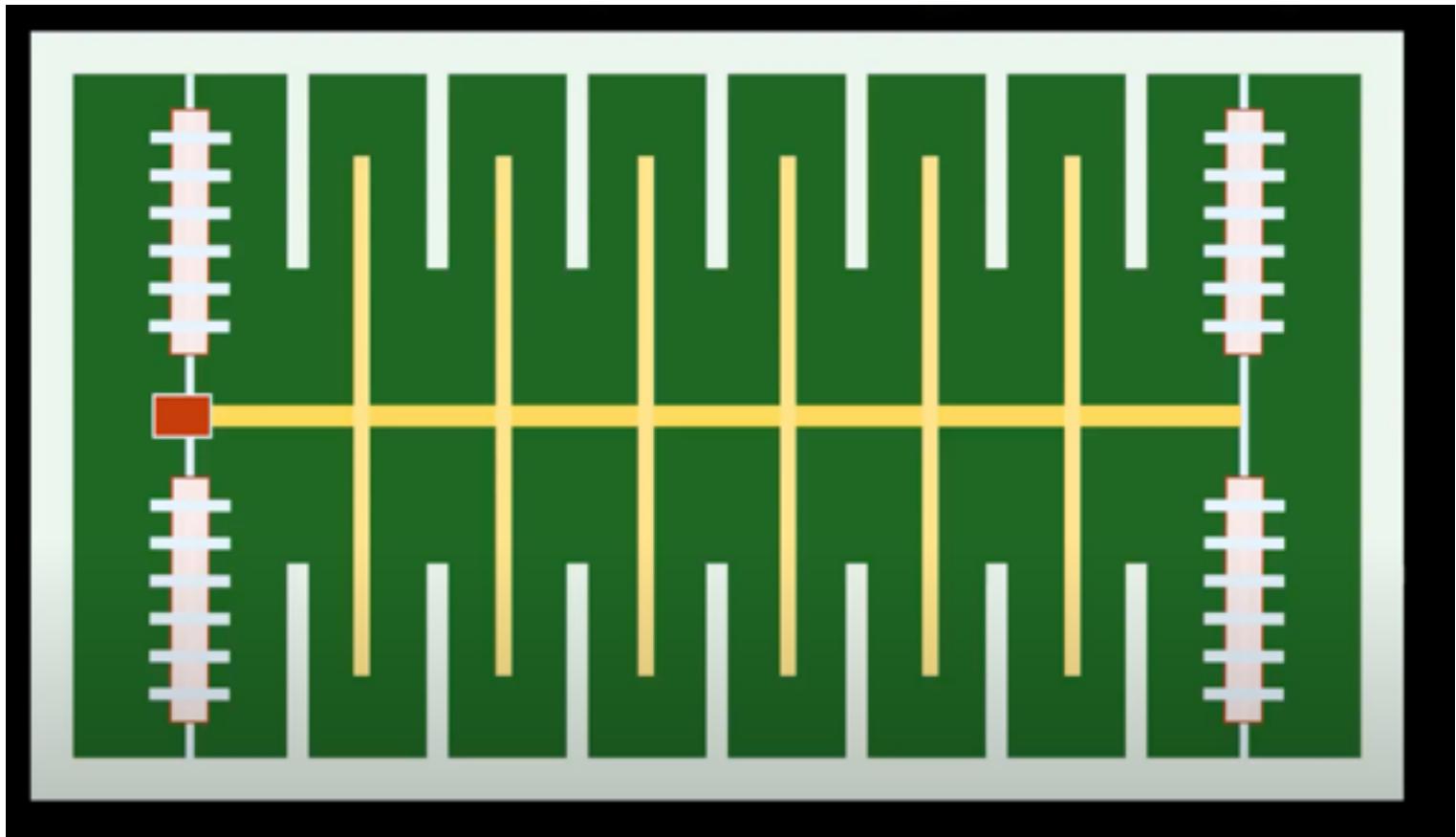
Capacitor



$$C = \epsilon \frac{\text{Area}}{x}$$

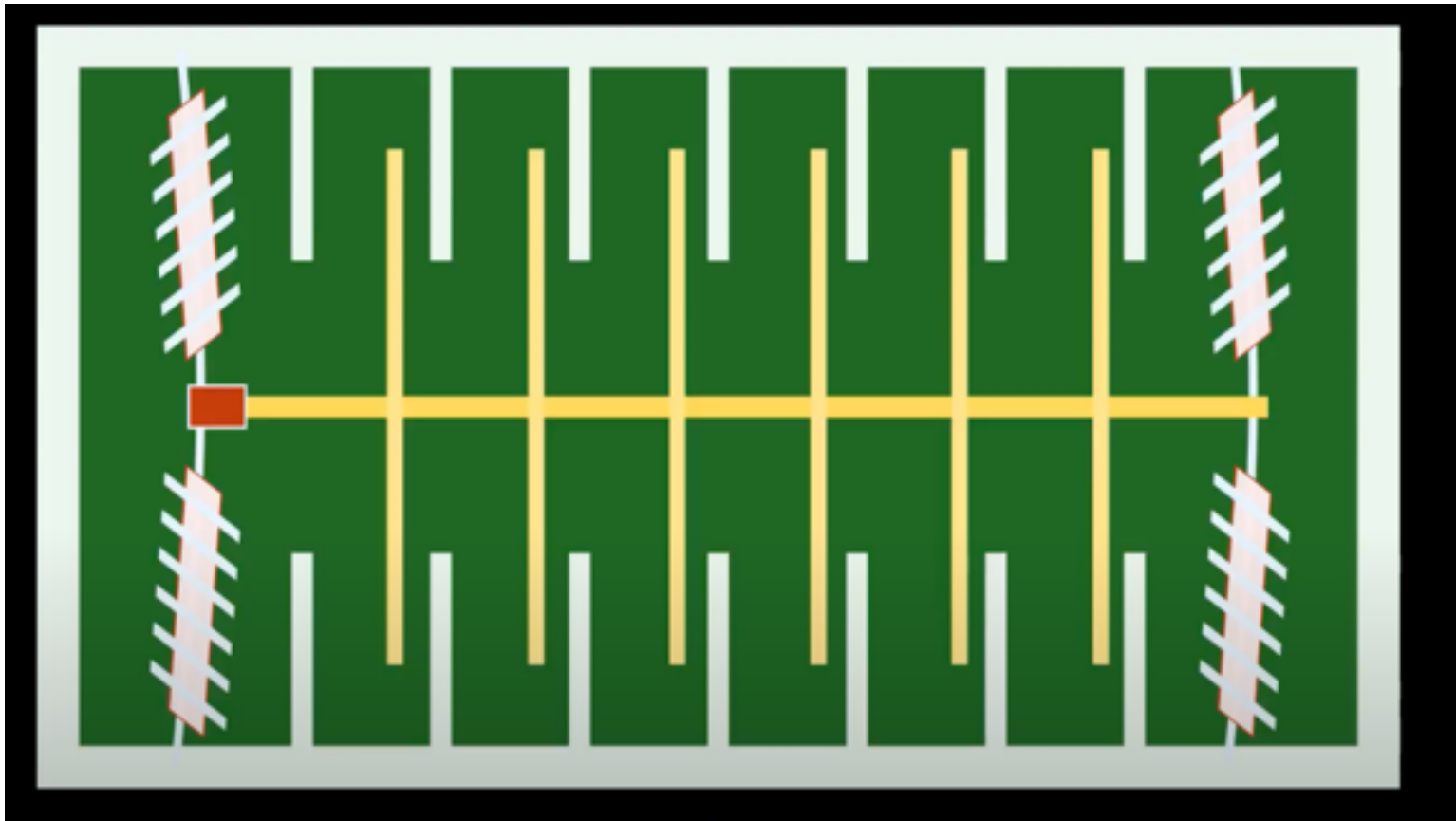
Measuring Displacement

- How do we measure displacement?
- Most common approach is to use capacitance and MEMS (Micro electro-mechanical systems)

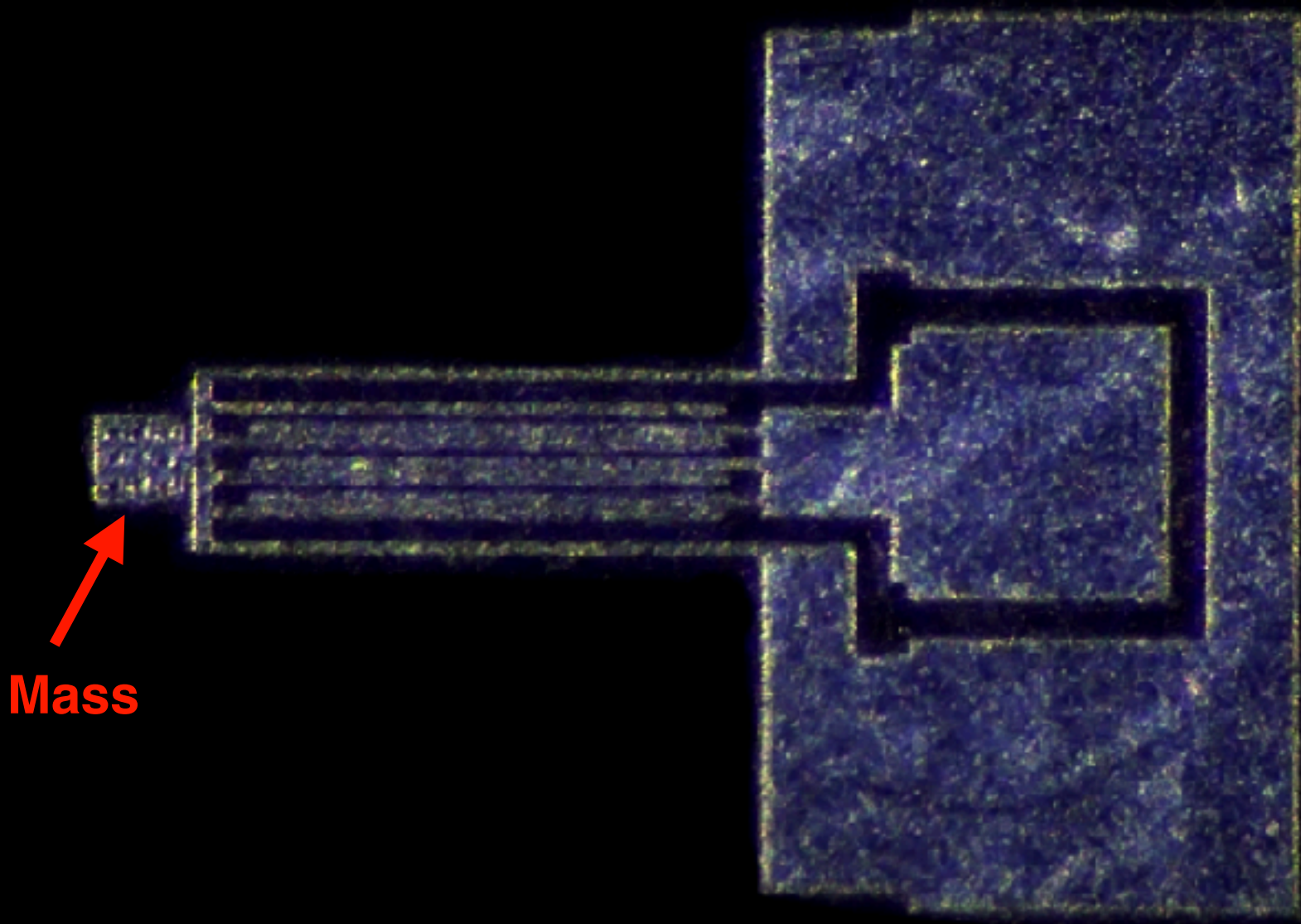


Measuring Displacement

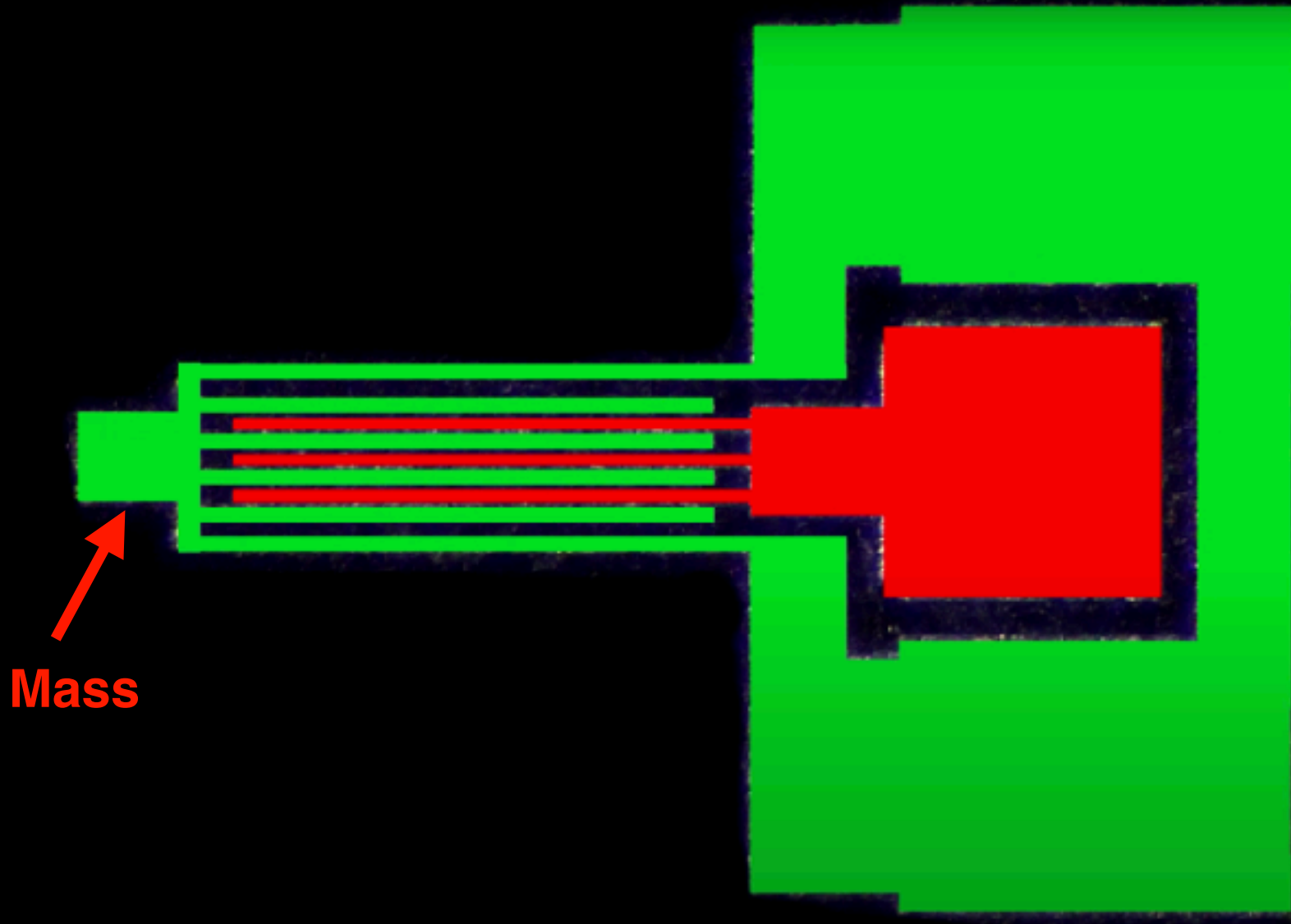
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MEMS Accelerometer

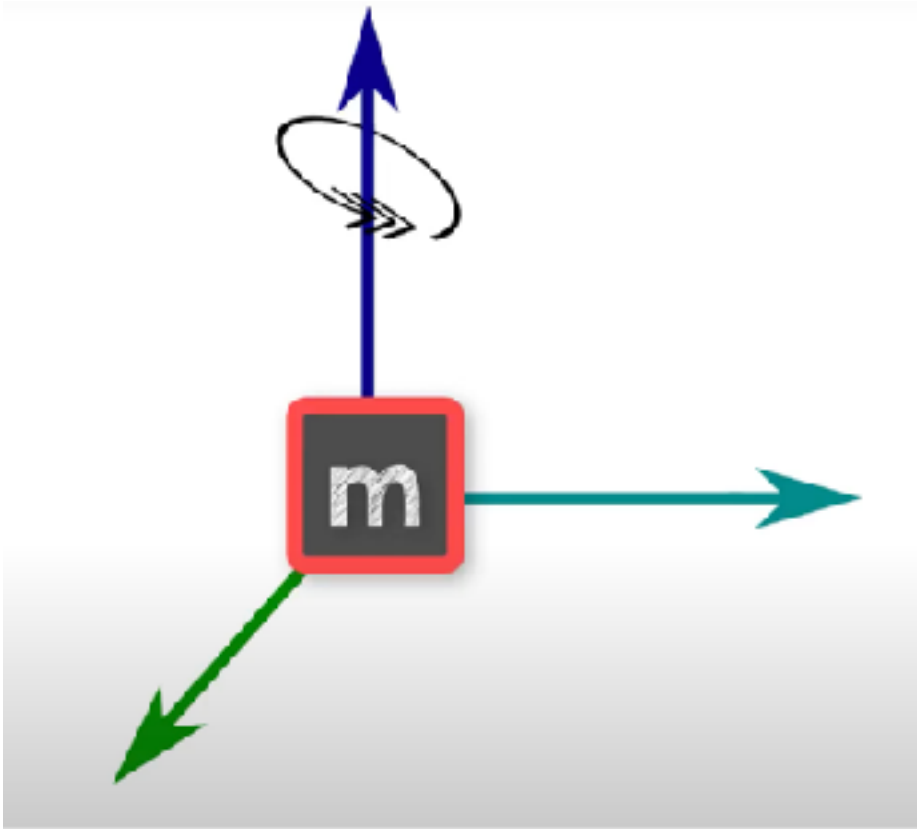


MEMS Accelerometer



How Gyroscopes Work?

The Coriolis Effect

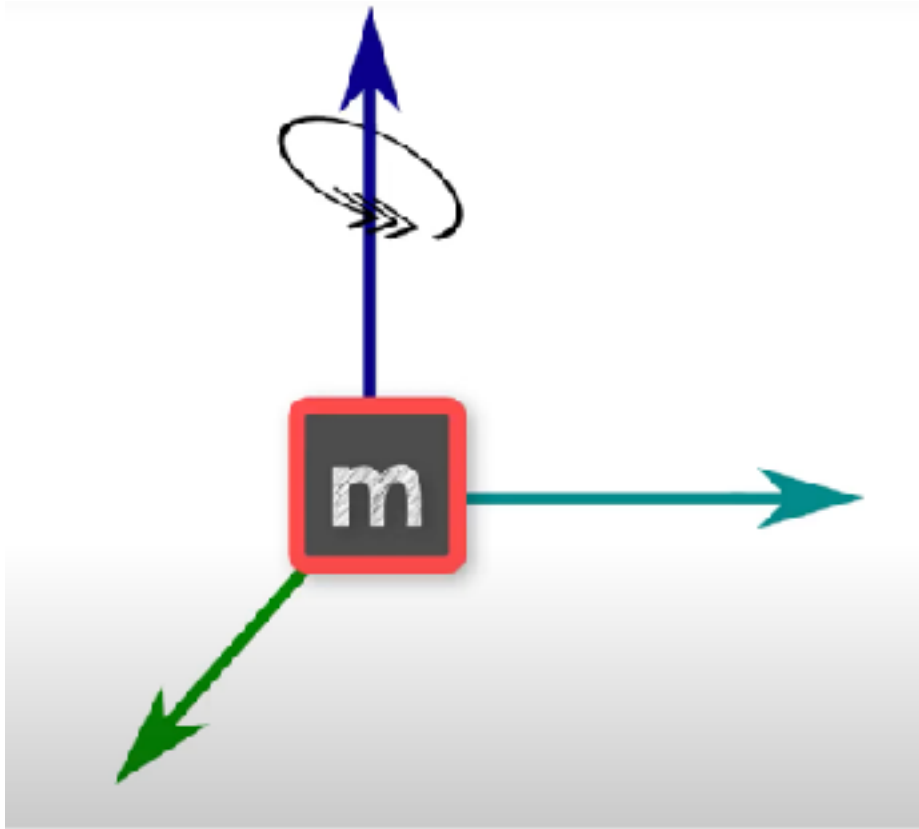


- Assume V_x
- Apply ω
- Experiences a fictitious force $F(\omega, V_x)$ following right hand rule

The Coriolis Effect

How Gyroscopes Work?

The Coriolis Effect

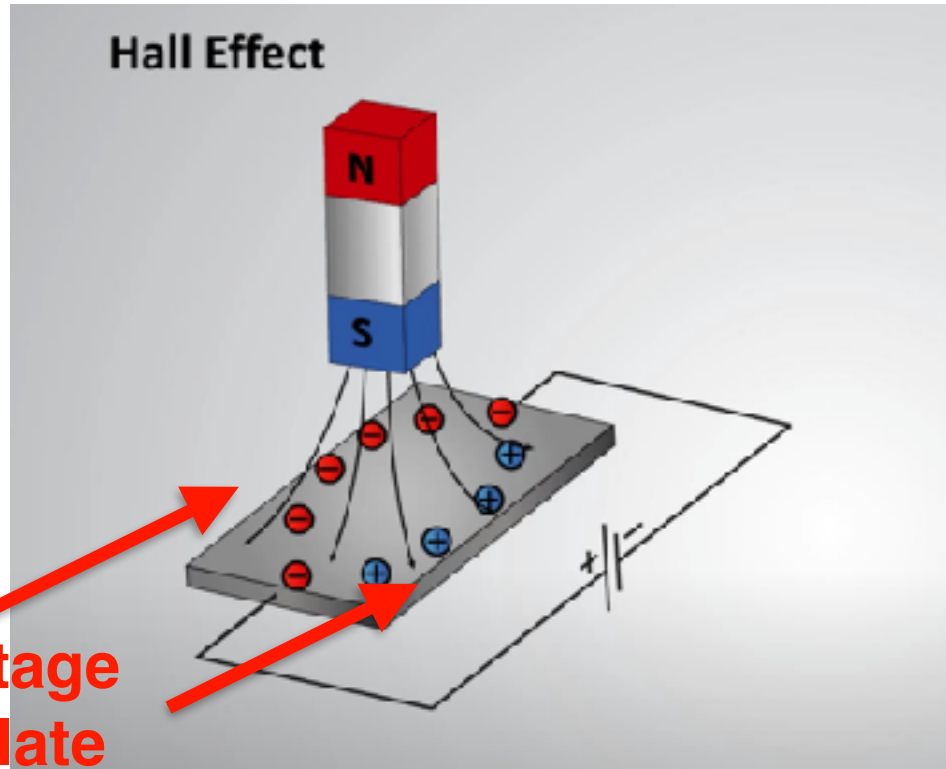


- Assume V_x
- Apply ω
- Experiences a fictitious force $F(\omega, V_x)$ following right hand rule

Can measure F in a similar fashion and use it to recover ω

How Magnetometers Work

- E.g., Compass
- Measure Earth's magnetic field



**Measure voltage
across the plate**

Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- **Understanding Sources of Errors**
- Dead reckoning by fusing multiple sensors

Gyroscope

Measured angular velocity:

$$\tilde{\omega} = \omega + b + \eta$$

True angular velocity

Bias

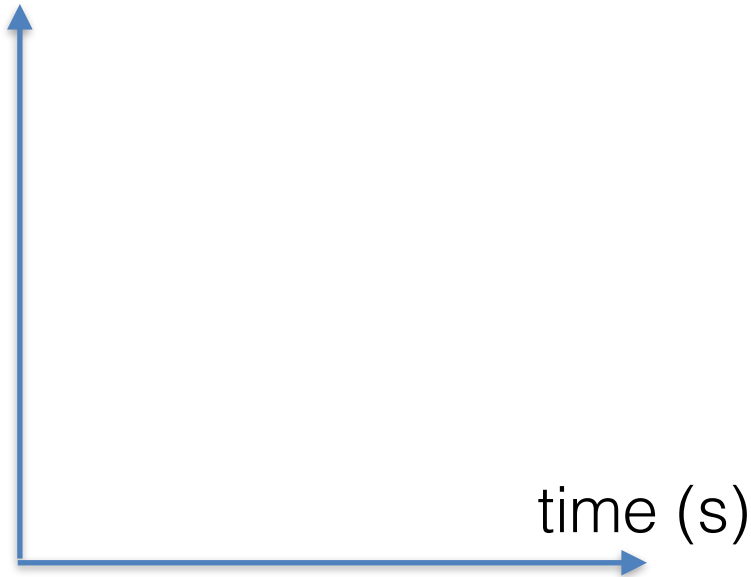
Noise (Gaussian, zero mean)

- How to get from angular velocity to angle?
 - Integrate, knowing initial position
- Linear integration? What are we missing?

Gyroscope- Some Math

Gyro Integration

Angle (degrees)

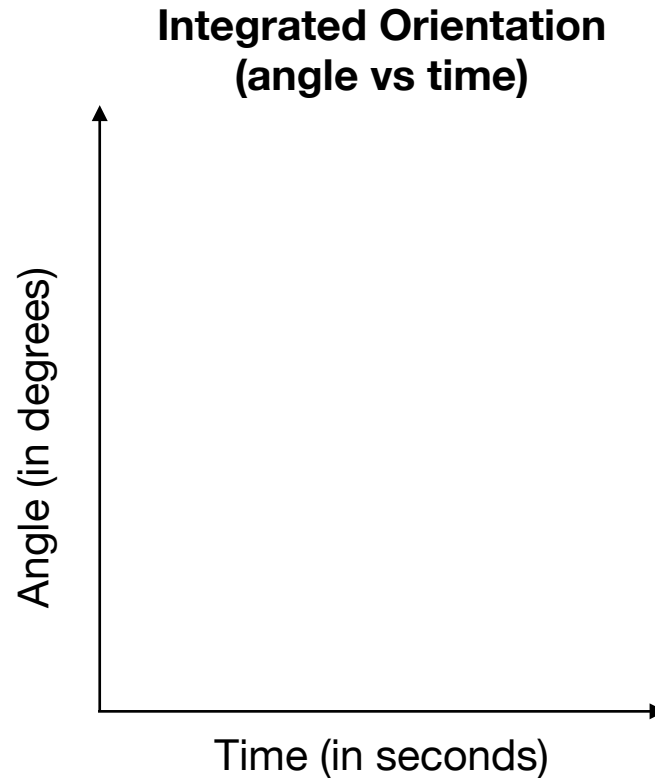
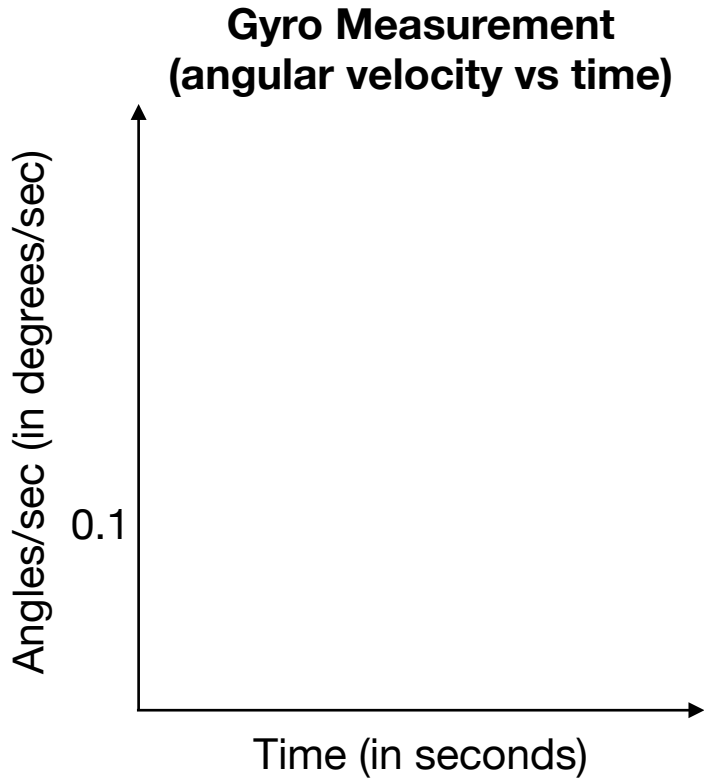


- Let's plot this for gyro measurement and for orientation
- Let's include ground truth and measured data for each

Consider:

- linear (angular) motion, no noise, no bias
- linear (angular) motion, with noise, no bias
- linear (angular) motion, no noise, bias
- nonlinear motion, no noise, no bias

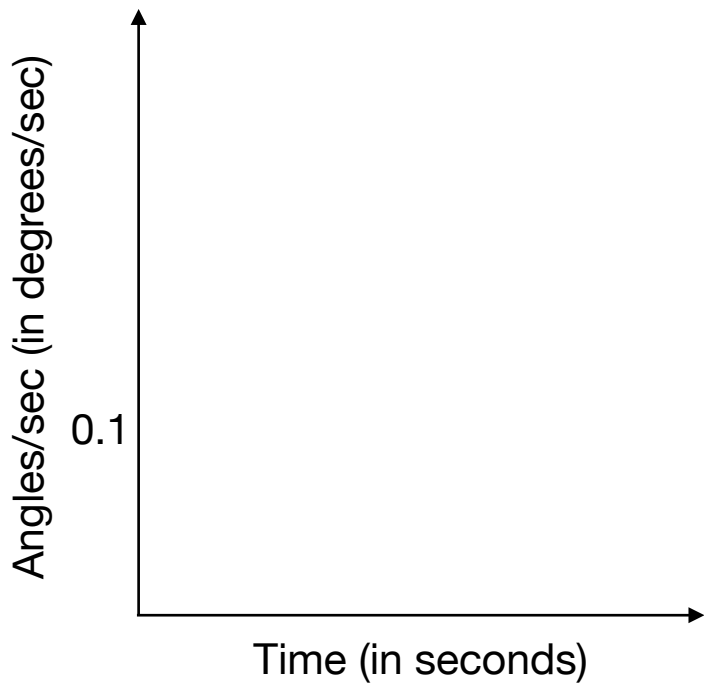
Gyro integration: linear motion, no noise, no bias



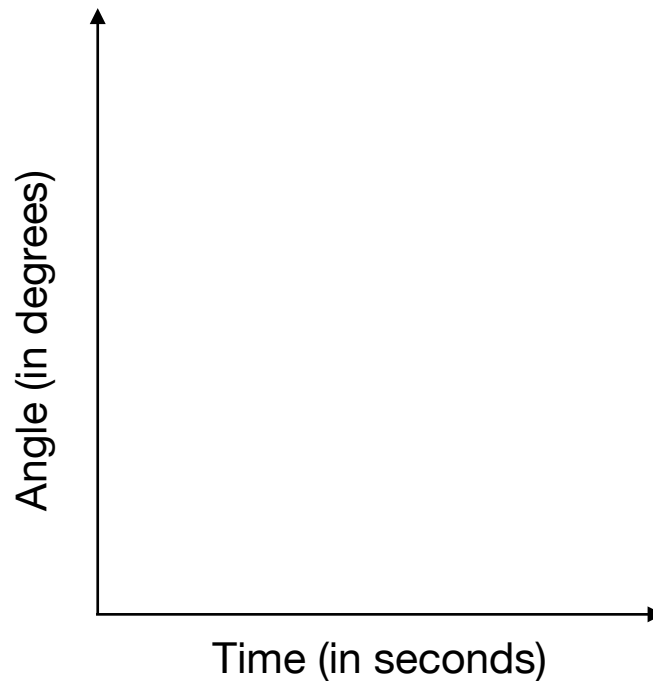
- Ground truth
- Measured/estimated angle

Gyro integration: linear motion, noise, no bias

Gyro Measurement

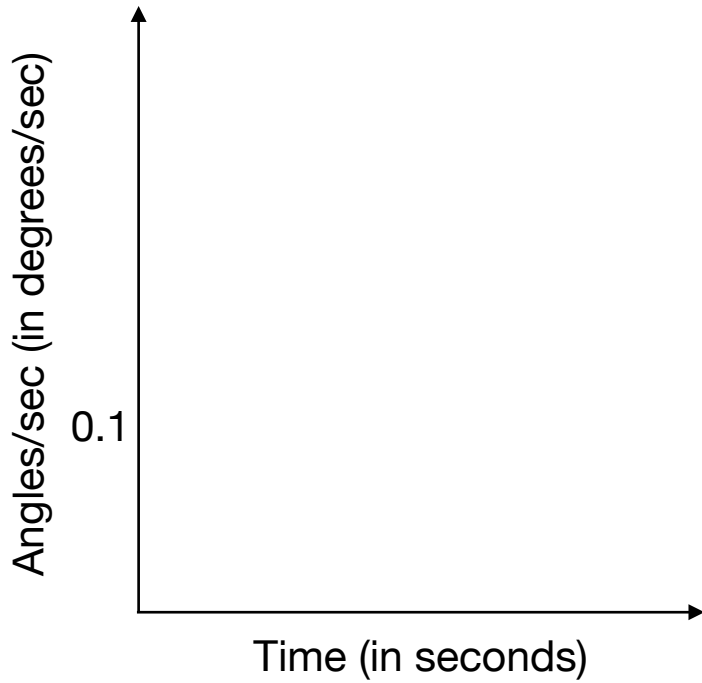


Integrated Orientation

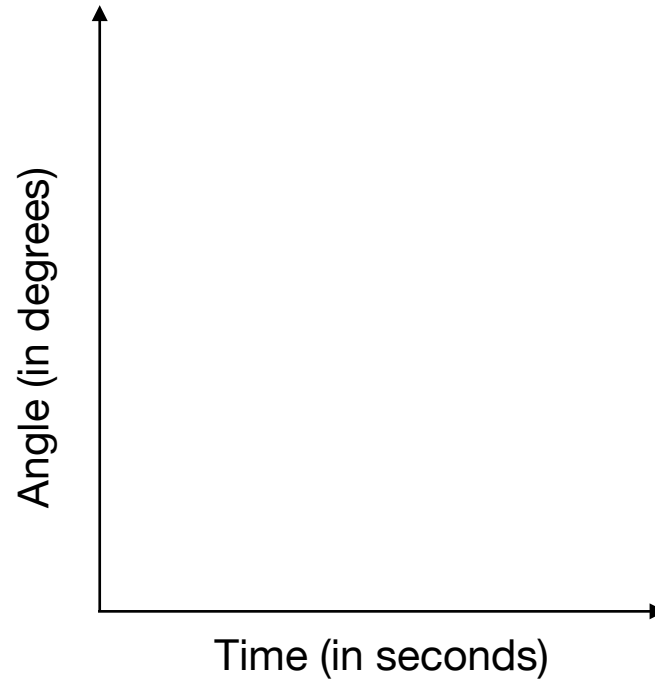


Gyro integration: linear motion, no noise, bias

Gyro Measurement

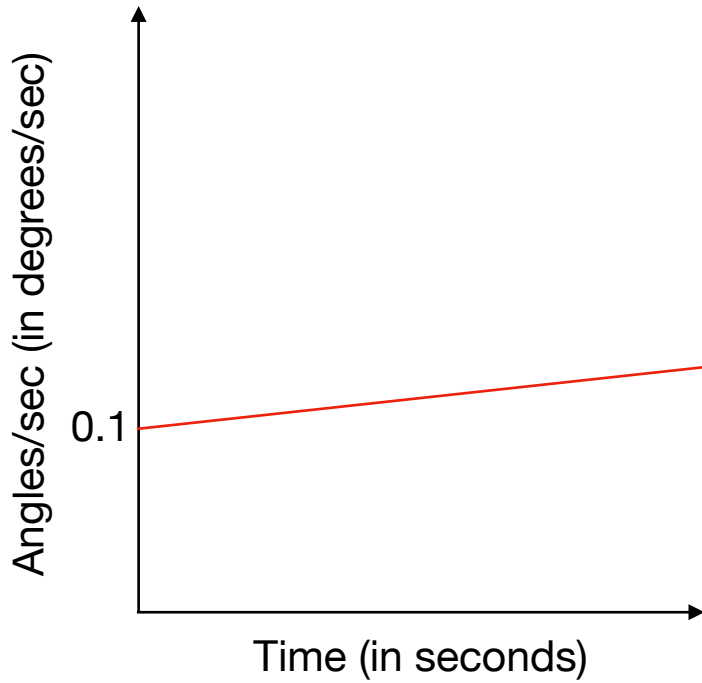


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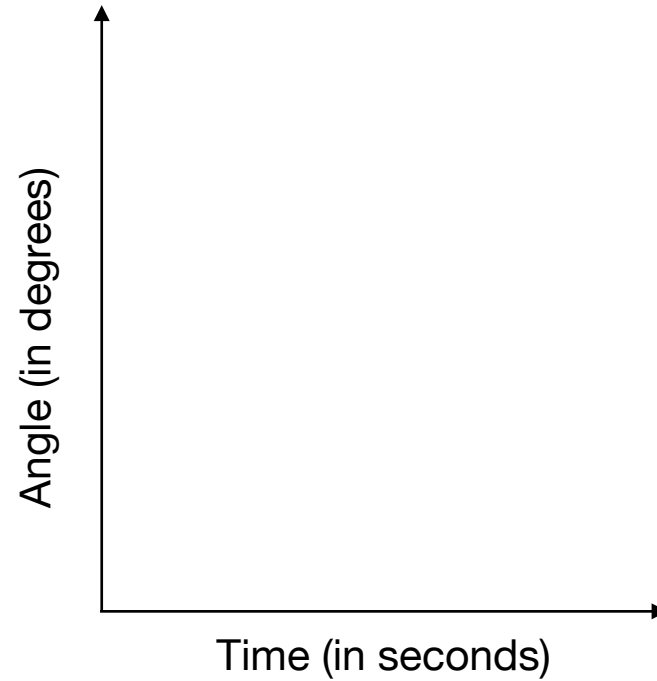


Gyro integration: nonlinear motion, no noise, no bias

Gyro Measurement



Integrated Orientation



Gyro Integration aka *Dead Reckoning*

- Works well for linear motion, no noise, no bias = unrealistic
- If bias is unknown and noise is zero -> drift (from integration)
- Bias and noise variance can be estimated, other sensor measurements used to correct for drift (sensor fusion)
- Accurate in short term, but not reliable in long term due to drift

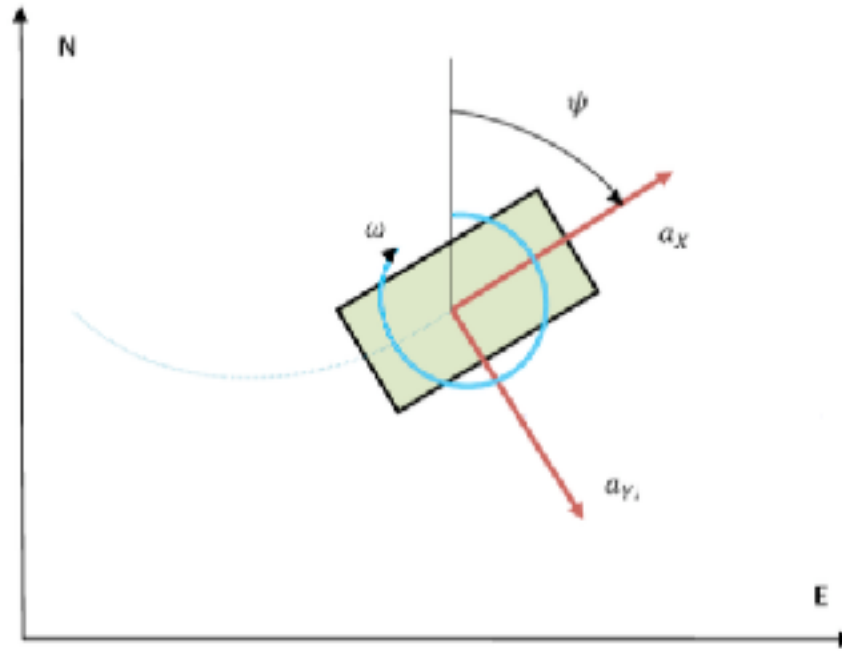
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Dead Reckoning

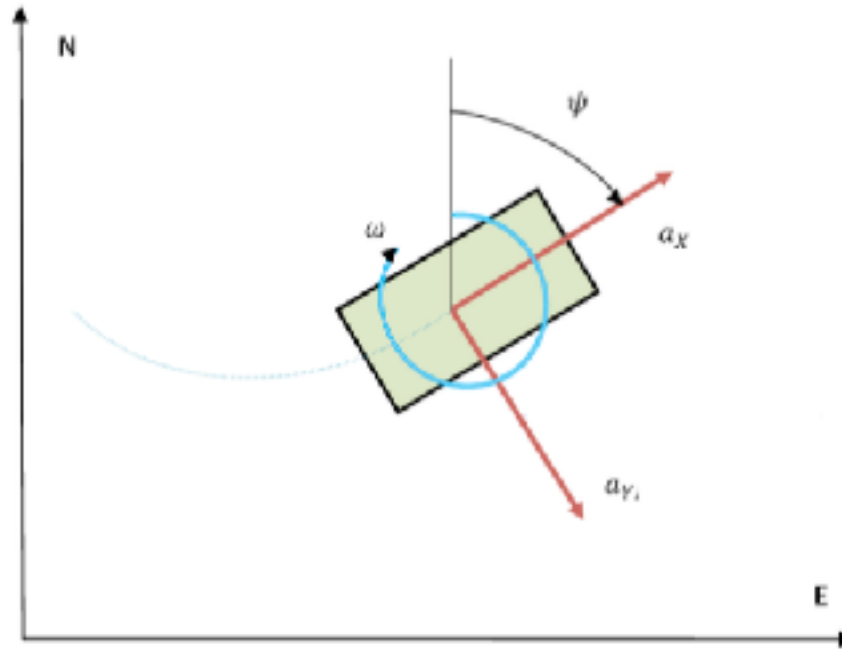
- The process of calculating one's current position by using a previously determined position, and advancing that position based upon known or estimated speeds over elapsed time and course
- Key things to keep in mind:
 - Frames of reference
 - Orientation change

2D Inertial Navigation in Strapdown System



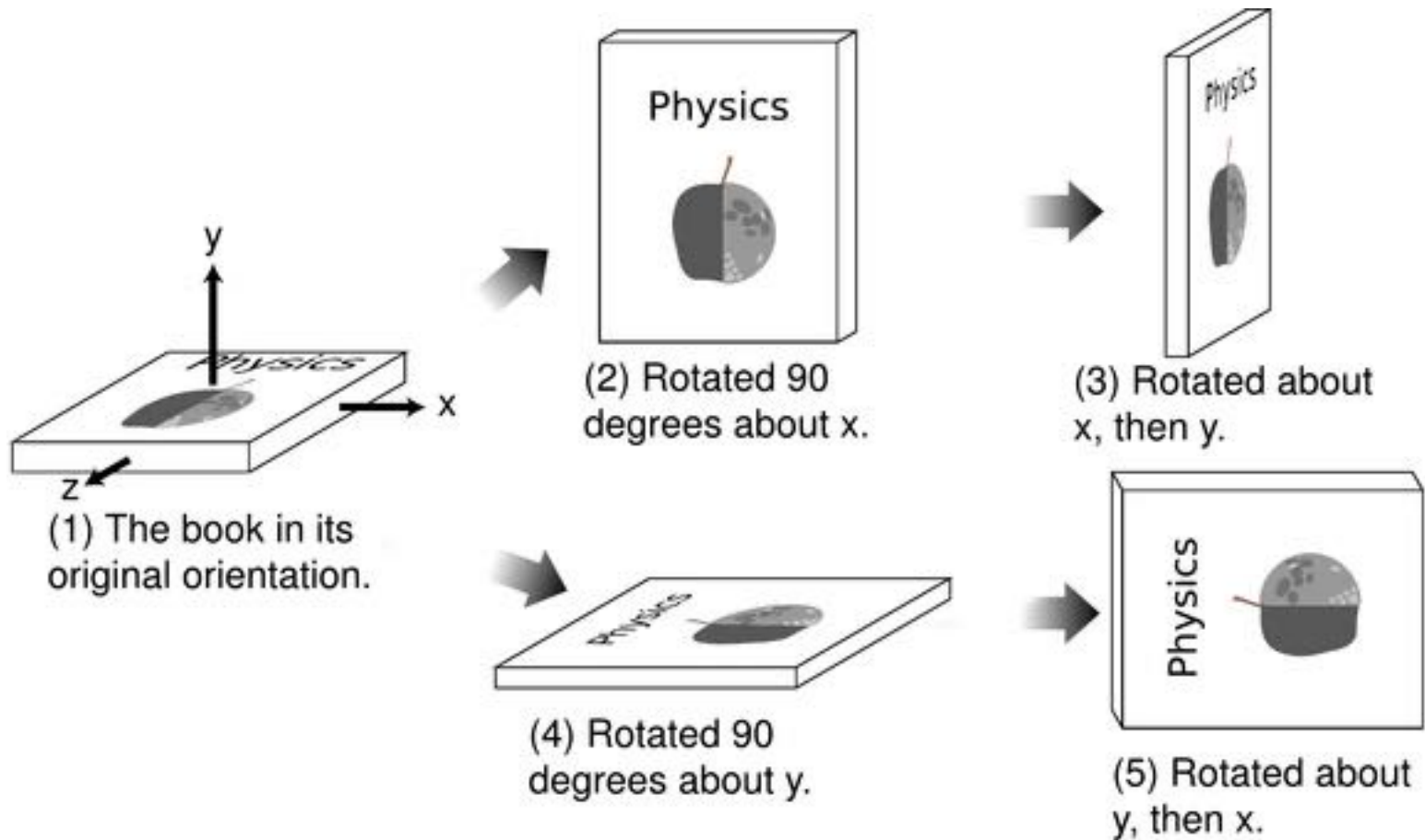
- Have a_x , a_y , ψ ; want E, N

2D Inertial Navigation in Strapdown System



2D Inertial Navigation in Strapdown System

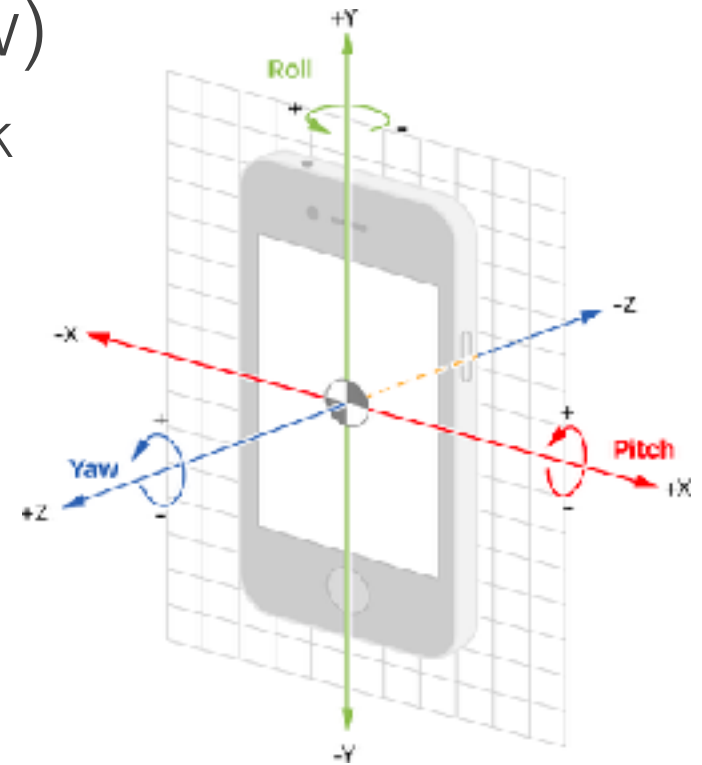
How about 3D Rotations?



Non-commutative = order matters!

3D Rotation Representations

- Rotation Matrix
 - 3 orthonormal vectors = 9 numbers
- Euler Angles (roll, pitch, yaw)
 - Symmetry problem, Gimbal lock
- Axis-angle
- Quaternions



Lecture Recap

- Importance of IMUs for navigation and sensing
- Coordinate systems and 6DOF
- IMU history and current use cases
- Basic principles of operation of different IMU sensors:
accelerometer, gyroscope, magnetometer
- Understanding Sources of Errors
- Dead reckoning by fusing multiple sensors

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